Total No. of Questions – [04]　　　　　Total No. of Printed Pages: 03

G.R. No.	

PAPER CODE	U111 – 201B

DECEMBER 2021 (INSEM+ ENDSEM) EXAM

F.Y. B. TECH. (SEMESTER - I)

COURSE NAME: CALCULUS

COURSE CODE: ES10201B

(PATTERN 2020)

Time: [2Hr]　　　　　[Max. Marks: 60]

(*) Instructions to candidates:

1) **Figures to the right indicate full marks.**
2) **Use of scientific calculator is allowed**
3) **Use suitable data where ever required**

Q.1 **Solve the following**

i) If $f(x,y) = \sin(xy) + x^2\log(y)$ then $\dfrac{\partial^2 f}{\partial y \partial x}$ at $\left(0,\dfrac{\pi}{2}\right)$ is [2]

a) 33　　b) 0　　c) 3　　d) 1

ii) If $u = x^3\sin^{-1}\left(\dfrac{y}{x}\right) + x^2y$ then value of $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$ at (4,4) is [2]

a) 128　　b) $64\left(\dfrac{\pi}{2}+1\right)$　　c) $128\left(\dfrac{\pi}{2}+1\right)$　　d) 64

iii) If $u = \tan^{-1}\left(\dfrac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \cdots$ [2]

a) $\dfrac{1}{2}\tan u$　　b) $\sin 2u$　　c) $\sin u\cos u$　　d) $\dfrac{1}{4}\sin 2u$

iv) If $x = r\cos\theta$ and $y = r\sin\theta$ then $\left(\dfrac{\partial r}{\partial x}\right)^2 + \left(\dfrac{\partial r}{\partial y}\right)^2 = \cdots$ [2]

a) 0　　b) 1　　c) r　　d) r^2

v) If $f(x,y) = x^3 + y^3 + 3axy$ then $\dfrac{dy}{dx} = \cdots$ [2]

a) $3x^2 + 3ay$　　b) $\dfrac{x^2+ay}{y^2-ax}$　　c) $\dfrac{x^2-ay}{y^2+ax}$　　d) $\dfrac{ay-x^2}{y^2-ax}$

vi) Minimum value of the function $x^2 + y^2 + 6x + 12$ is [2]

a) 0　　b) 3　　c) -3　　d) 6

vii) If $x = r\cos\theta$ and $y = r\sin\theta$ then $\dfrac{\partial(r,\theta)}{\partial(x,y)}$ is [2]

a) 1 b) r c) $\dfrac{1}{r}$ d) 0

viii) Percentage error in calculating area of an ellipse having an error of 1% and 2% made in major and minor axis respectively is [2]

a) 1% b) 2% c) 3% d) – 1%

ix) The function $x^2 + 4y^3 - 12y^2 - 36y + 2$ has at point $(0, -1)$ [2]

a) Maxima b) Minima c) Saddle point d) None of these

x) If $u = \dfrac{x+y}{1-xy}$ and $v = \tan^{-1}x + \tan^{-1}y$ then $\dfrac{\partial(u,v)}{\partial(x,y)}$ is [2]

a) 1 b) $\dfrac{1+y^2}{(1-xy)^2}$ c) $\dfrac{1+x^2}{(1-xy)^2}$ d) 0

xi) $\displaystyle\int_0^{2\pi} \sin^7 x\, dx = \cdots$ [2]

a) $\dfrac{32\pi}{35}$ b) $\dfrac{64}{35}$ c) $\dfrac{32}{35}$ d) 0

xii) $\displaystyle\int_0^{\infty} e^{-x} x^{5/2}\, dx = \cdots$ [2]

a) $\dfrac{15\sqrt{\pi}}{8}$ b) $\dfrac{3\sqrt{\pi}}{8}$ c) $\dfrac{3\sqrt{\pi}}{4}$ d) $\dfrac{3\sqrt{\pi}}{2}$

xiii) $\displaystyle\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta = \cdots$ [2]

a) $\dfrac{\pi^2}{2}$ b) $\sqrt{\pi}$ c) $\dfrac{\pi}{\sqrt{2}}$ d) $\dfrac{\sqrt{\pi}}{2}$

xiv) The value of a_n in the Fourier series of $f(x) = x$ in the the interval $0 < x < 2\pi$ is [2]

a) $\dfrac{2}{n}$ b) $-\dfrac{2}{n}$ c) $\dfrac{1}{n}$ d) 0

xv) The value of a_0 in the Fourier series of $f(x) = x^2$ in the the interval $-\pi < x < \pi$ is [2]

a) $\dfrac{\pi^2}{3}$ b) $\dfrac{2\pi^2}{3}$ c) π^2 d) 0

Q.2 **Solve any two out of three**

a) Solve $\frac{2x}{y^3}dx + \left(\frac{y^3 - 3x^2}{y^4}\right) dy = 0$ [5]

b) Solve $(1 + y^2)dx = (\tan^{-1}y - x)dy$ [5]

c) Find an orthogonal trajectory of family of curves given by $r^2 = a \sin 2\theta$. [5]

Q.3 **Solve any two out of three.**

a) Trace the curve $y(x^2 - 1) = x^2 + 1$. [5]

b) Trace the curve $r^2 = a^2 \cos 2\theta$. [5]

c) Trace the curve $x^{2/3} + y^{2/3} = a^{2/3}$. [5]

Q.4 **Solve any two out of three.**

a) Evaluate $\int_0^1 \int_y^{1+y^2} x^2 y \, dxdy$. [5]

b) Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x \, dzdxdy$. [5]

c) Find the area enclosed by the parabolas $x^2 = 4ay$ and $y^2 = 4ax$. [5]