Total No. of Questions – [4] Total No. of Printed Pages: 04

G.R. No.	

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DECEMBER 2021 (INSEM+ ENDSEM) EXAM

F.Y. B. TECH. (SEMESTER - I)

COURSE NAME: LINEAR ALGEBRA

COURSE CODE: ES10201A

(PATTERN 2020)

Time: [2Hr] [Max. Marks: 60]

(*) Instructions to candidates:

1) **Figures to the right indicate full marks.**
2) **Use of scientific calculator is allowed**
3) **Use suitable data where ever required**

Q.1 **Solve the following**

i) Rank of the matrix A= $\begin{bmatrix} 4 & 4 & -4 \\ 0 & 4 & 4 \\ 0 & 0 & 4 \end{bmatrix}$ is [2]

A] 1 B] 2 C] 3 D] 0

ii) Non zero *Solution of*
$x + y + z = 0, 2x + 2y + 2z = 0 , 3x + 3y + 3z = 0$ *is* [2]

$A)\ x = t_1 + t_2\ , y = t_1\ , z = t_2$
$B)\ x = t_1 + 2t_2\ , y = t_1\ , z = t_2$
$C)\ x = t_1 - t_2\ , y = t_1\ , z = t_2$
$D)\ x = -t_1 - t_2\ , y = t_1\ , z = t_2$

iii In solving the system of equations AX = B if $\rho(A) \neq \rho([A: B])$. [2]
Then given system has

A] Unique solution B] No Solution
C] One free parameter solution D] Two free parameter solutions

iv) Rank of the matrix A= $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is [2]

A] 0 B] 1 C] 2 D] 3

v) Solution of the homogenous system x+y=0 and x-2y=0 is [2]

A] No solution B] x = 0, y = 0 C] x=1 y=1 D] x=2 ,y=0

vi) Which of the following set is subspace of $\mathbb{R}^2$? [2]

A] W= { (x , y) / y=2x+3 }

B] W= { (x, y) / x = 3 }

C] W= { (x , y) / y=2x }

D] W= { (x , y) / y = 2 }

vii) Linear Span of vectors v1= (1 ,1) and V2 = (1, 2) is [2]

A] One dimensional Subspace of $\mathbb{R}^3$

B] Two dimensional Subspace of $\mathbb{R}^3$

C] Three dimensional Subspace of $\mathbb{R}^3$

D] Zero dimensional Subspace of $\mathbb{R}^3$

viii)Let V be vector space of set of all polynomials of degree ≤ 2 [2]

V= $\{a_0 + a_1 t \,/\, a_0 \,, a_1, a_2, a_3 \in \mathbb{R}\}$ then Basis of V are

A] $\{1 \;\; t\}$ B] $\{t\}$

C] $\{1, \; t \;, t^2 \;\; t^3\}$ D] $\{0 \;, \; t \;, \; t^2\}$

ix) Dimensions of the row space of the matrix A = $\begin{bmatrix} 1 & 3 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$ are [2]

A] 1 B] 2 C] 3 D] 4

x) Basis of the Column space of the matrix A = $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix}$ are [2]

A] $\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix} \}$

B] $\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \end{bmatrix} \}$

C] $\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix} \}$

D] A] $\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix} \}$

xi) Which of the following is Linear Transformation from $\mathbb{R}^3 \dashrightarrow \mathbb{R}^2$? [2]

A] $T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x+y \\ 3x-7y+3z \end{bmatrix}$ B] $T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} xy \\ z \end{bmatrix}$

C] $T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x+y \\ x+yz \end{bmatrix}$ D] $T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x-y+2 \\ 3 \end{bmatrix}$

xii) Consider the Linear Transformation A : $\mathbb{R}^3 \longrightarrow \mathbb{R}^3$ define as AX= Y [2]

Where A= $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ then dimensions of Im A are

A] 1 B] 2 C] 3 D] 4

xiii) Consider the Linear Transformation A : $\mathbb{R}^3 \longrightarrow \mathbb{R}^3$ define as AX= Y [2]

Where A= $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ then dimensions of Kernel A are

A] 1 B] 2 C] 3 D] 4

xiv) Linear Transformation Y = AX where A= $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ is [2]

A] Regular B] Orthogonal C] Singular D] Composite

xv) Linear Transformation Y = AX where A= $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 1 & 1 & 3 \end{bmatrix}$ is [2]

A] Regular B] Orthogonal C] Singular D] Composite

Q2 **Solve any two out of three**

a) Apply the Gram-Schmidt orthogonalization process to find orthogonal basis, of the vectors S= $\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix} \right\}$ [5]

b) . Let V be a vector space of polynomials with inner product $\langle f(t) , g(t) \rangle = \int_{-1}^{1} f(t)g(t)dt$ Apply the Gram-Schmidt orthogonalization process to $S = \{ 1, \ t, \ t^2 \}$ to find orthogonal basis. [5]

c) Let P(t) be vector space of polynomials with inner product $\langle f(t) , g(t) \rangle = \int_{0}^{1} f(t)g(t)dt$ then for $f(t) = t^2$ $\& \, g(t) = t - 3$ [5]

find i) $||f||$ ii) $||g||$

Q.3 **Solve any two out of three**

a) Find all Eigen values and Eigen vectors of the matrix A= $\begin{bmatrix} 5 & 6 \\ 3 & -2 \end{bmatrix}$ [5]

b) Check whether or not A is diagonalizable and if yes then diagonalize it , where A= $\begin{bmatrix} 5 & -1 \\ 1 & 3 \end{bmatrix}$ [5]

c) Verify Caley-Hamilton Theorem for the matrix [5]

A= $\begin{bmatrix} -1 & 8 & 7 \\ 0 & -8 & 0 \\ 0 & 0 & -3 \end{bmatrix}$ & use it to find A^{-1}

Q.4 **Solve any two out of three**

a) Find the symmetric matrix that corresponds to the following quadratic form and hence determine the nature of the quadratic form [5]

q(x y z) = $2x^2 + 4xy + 2y^2 + z^2$

b) Find Signature of the quadratic form [5]

Q(x,y,z)= $3x^2 + 4xy - 2xz + 3y^2 - 2yz + 4z^2$

c) Using orthogonal digitalization find Canonical form corresponding to quadratic form Q(x,y,z)= $3x^2 + 3y^2 + 6xy$ [5]

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