Total No. of Questions – [4] Total No. of Printed Pages: 04

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DECEMBER 2021 (INSEM+ ENDSEM) EXAM

F.Y. B. TECH. (SEMESTER - I)

COURSE NAME: LINEAR ALGEBRA

COURSE CODE: ES10201A

(PATTERN 2020)

Time: [2Hr] [Max. Marks: 60]

(*) Instructions to candidates:

1) **Figures to the right indicate full marks.**
2) **Use of scientific calculator is allowed**
3) **Use suitable data where ever required**

Q.1 **Solve the following** [2]

i) Rank of the matrix A= $\begin{bmatrix} 1 & 3 & -1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ is

A] 1 B] 2 C] 3 D] 0

ii) Non zero *Solution of* [2]

$x + y + z = 0, 2x + 2y + 2z = 0, 3x + 3y + 3z = 0$ *is*

$A)\ x = t_1 + t_2, y = t_1, z = t_2$
$B)\ x = t_1 + 2t_2, y = t_1, z = t_2$
$C)\ x = t_1 - t_2, y = t_1, z = t_2$
$D)\ x = -t_1 - t_2, y = t_1, z = t_2$

iii) In solving the system of equations AX = B if $\rho(A) = \rho([A:B\}) = 1$ and having 3 number of unknown variables. Then given system has [2]

A] Unique solution B] No Solution
C] One free parameter solution D] Two free parameter solutions

iv) Rank of the matrix A= $\begin{bmatrix} 2 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$ is [2]

A] 1 B] 2 C] 3 D] 0

v) If A is non singular matrix then Homogenous system of equation AX=0 has [2]

A] Only Trivial solution B] Non trivial solutions

C] No solutions D] None of above

vi) Which of the following set is subspace of $\mathbb{R}^2$? [2]

A] W= { (x , y) / y=2x+3 }

B] W= { (x, y) / x = 3 }

C] W= { (x , y) / y=2x }

D] W= { (x , y) / y = 2 }

vii) Linear Span of vectors v1= (1 ,0, 0) and V2 = (0, 0, 1) is [2]

A] One dimensional Subspace of $\mathbb{R}^3$

B] Two dimensional Subspace of $\mathbb{R}^3$

C] Three dimensional Subspace of $\mathbb{R}^3$

D] Zero dimensional Subspace of $\mathbb{R}^3$

viii) Let V be vector space of set of all polynomials of degree $\leq$ 3 [2]

V= $\{a_0 + a_1t + a_2t^2 + a_3t^3 \ / \ a_0 \ , a_1, a_2, a_3 \in \mathbb{R}\}$ then Basis of V are

A] $\{1 \ \ t\}$ B] $\{ \ t \ \}$

C] $\{ \ 1, \ t \ , t^2 \ \ t^3\}$ D] $\{0 \ , \ t \ , \ t^2\}$

ix) Dimensions of the row space of the matrix A = $\begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$ are [2]

A] 1 B] 2 C] 3 D] 4

x) Basis of the Column space of the matrix A = $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix}$ are [2]

A] $\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix} \}$

B] $\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \end{bmatrix} \}$

C] $\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix} \}$

D] A] $\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix} \}$

xi) Which of the following is Linear Transformation from $\mathbb{R}^3 \longrightarrow \mathbb{R}^2$? [2]

A] T(x y z)= (x +y , xyz)

B] T(x y z)= (x -z y+ z)

C] T(x y z)= (xy yz)

D] T(x y z)= (x+z , xy)

xii) Consider the Linear Transformation A : $\mathbb{R}^3 \longrightarrow \mathbb{R}^3$ define as AX= Y [2]

Where A= $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ then dimensions of Im A are

A] 1 B] 2 C] 3 D] 4

xiii) Consider the Linear Transformation $A : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ define as AX= Y [2]

Where A= $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ then dimensions of Kernel A are

A] 1 B] 2 C] 3 D] 4

xiv) Linear Transformation Y = AX where A= $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ is [2]

A] Regular B] Orthogonal C] Singular D] Composite

xv) Linear Transformation Y = AX where A= $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 1 & 1 & 3 \end{bmatrix}$ is [2]

A] Regular B] Orthogonal C] Singular D] Composite

Q2 **Solve any two out of three**

a) . Apply the Gram-Schmidt orthogonalization process to find orthogonal basis , [5]

of the following set of vectors $S=\left\{V1 = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}, V2 = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, V3 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}\right\}$

b) Let V be a vector space of polynomials with inner product [5]

$\langle f(t) , g(t) \rangle = \int_{-1}^{1} f(t)g(t)dt$ Apply the Gram-Schmidt orthogonalization process

on $S = \{V_1 = 1, \ V_2 = t, \ V_3 = t^2 \}$ to find orthogonal basis.

c) Let P(t) be vector space of polynomials with inner product [5]

$\langle f(t) , g(t) \rangle = \int_{0}^{1} f(t)g(t)dt$ then for $f(t) = t^2 \quad \& \, g(t) = t - 3$

find i) $\langle f , g \rangle$ ii) ||f||

Q.3 **Solve any two out of three**

a) Q1] Find all Eigen values and Eigen vectors of the matrix [5]

A= $\begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}$

b) Check whether A is diagonalizable & if yes diagonalize it where A= $\begin{bmatrix} 3 & -4 \\ 2 & -6 \end{bmatrix}$ [5]

c) Verify Caley-Hamilton Theorem for the matrix

[5]

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$ & use it to find A^{-1}

Q.4 **Solve any two out of three**

a) Find the symmetric matrix that corresponds to the following quadratic form and hence determine the nature of the quadratic form [5]

q(x y z) = $2x^2 - 2xz + 2y^2 + 2z^2$

b) Find Signature of the quadratic form [5]

Q(x,y,z) = $2x^2 + 4xy + 2y^2 + z^2$

c) Using orthogonal digitalization find Canonical form corresponding to quadratic form Q(x,y,z)= $2x^2 - 4xy + 5y^2$ [5]

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