DECEMBER 2021 / INSEM+ENDSEM
F. Y. M. TECH. (COMPUTER ENGINEERING) (SEMESTER – I)
COURSE NAME: MATHEMATICAL FOUNDATION OF COMPUTER SCIENCE
COURSE CODE: CSPA11201
(PATTERN 2020)

Time: [3 Hours] [Max. Marks: 60]

(*) Instructions to candidates:
1) All Questions are compulsory
2) Figures to the right indicate full marks.
3) Use of scientific calculator is allowed
4) Use suitable data wherever required

Q1 a. When is a digraph an Euler digraph? Draw an Euler digraph. (05)
b. Prove that every connected graph has at least one spanning tree (05)

Q2 a. When looking at a person's eye colour, it turns out that 1% of people in the world has green eyes Consider a group of 20 people. Find (05)
i) Nine have green eyes.
ii) At the most three have green eyes.

b. Let X be the continuous random variable with density function (05)

$$f(x) = \begin{cases} x \times e^{-x} & 0 \le x \le 5 \\ 0 & otherwise \end{cases}$$

a) Find probability P(1<= X <=2)
b) Find probability P(X<=3)

Q3 a. Toss an unbiased die that has six sides. Observe the number that comes on top. Calculate the expected value and variance (05)

b. Find expected value and variance of X (05)

$$f(x) = \begin{cases} \frac{3}{4}x(2-x) & 0 < x < 2 \\ 0 & otherwise \end{cases}$$

Also find the variance for babies' distribution

Q4 The heights and weights of a sample of 11 students are: (10)

Height (m) h	1.36	1.47	1.54	1.56	1.59	1.63	1.66	1.67	1.69	1.74	1.81
Weight (kg) w	52	50	67	62	69	74	59	87	77	73	67

a) Calculate the regression line of w on h.
b) Use the regression line to estimate the weight of someone whose height is 1.6m.
c) Coefficient of determination

Q5 a. In a city, during the first 3 months of a year (05)

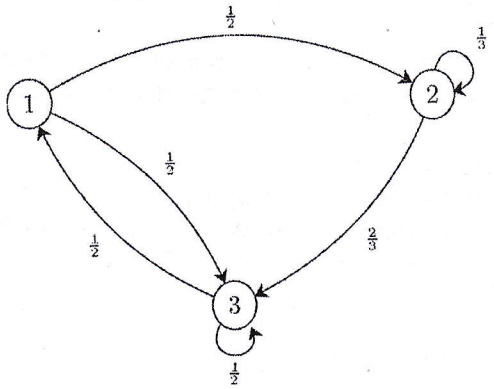
		Weather on next day	
Weather on one day		Dry	Wet
	Dry	57	12

	Wet	12	8

Find the probability that it will be dry two days after a wet day
Find the probability that it will be dry 9 days after a wet day

b. Patients arrive at the doctor's office according to a Poisson process with rate λ = 1/10 minute. The doctor will not see a patient until at least three patients are in the waiting room. (05)
(a) Find the expected waiting time until the first patient is admitted to see the doctor.
(b) What is the probability that nobody is admitted to see the doctor in the first hour?

Q6 Consider a continuous-time Markov chain X(t) with the jump chain shown in Figure Assume λ1=2, λ2=3 and λ3=4. (10)



Find the stationary distribution of the jump chain
Using the stationary distribution of the jump chain, find the stationary distribution for X(t).