Dec-2022

Total No. of Questions – [4]

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G.R. No.	

PAPER CODE	U112-201A(BE)

Dec 2022 (INSEM+ ENDSEM) EXAM
F.Y. B. TECH. (SEMESTER - II)
COURSE NAME: Linear Algebra
COURSE CODE: ES10201A
(PATTERN 2020)

Time: [2Hr]

[Max. Marks: 60]

(*) Instructions to candidates:

1) **Figures to the right indicate full marks.**
2) **Use of scientific calculator is allowed**
3) **Use suitable data where ever required**

Q.1 **Solve the following**

i) Rank of the matrix A of order 3X4 is [2]

A] Equal to 3

B] Equal to 4

C] Less than or equal to 4

D] Less than or equal to 3

ii) Non zero *Solution of* $x + y + z = 1, y + z = 1, 3y + 4z = 3$ *is* [2]

A] x=0 ,y= 0 & z=0

B] x=1 ,y= 1 & z=0

C] x=0 ,y= 1 & z=0

D] x=1-2t ,y= 1-t & z=t

iii) In solving the system of equations AX = B if $\rho(A) = 2 \,\&\, \rho([A:B\}) = 3$ and having 3 number of unknown variables. Then given system has [2]

A] Unique solution

B] No Solution

C] One free parameter solution

D] Two free parameter solutions

iv) Rank of the matrix A= $\begin{bmatrix} 2 & 1 \\ 4 & 2 \\ 6 & 4 \end{bmatrix}$ is [2]

A] 1 B] 2 C] 3 D] 0

v) If A is non-singular matrix then Homogenous system of equation AX=0 has

A] Only Trivial solution

B] Non trivial solutions

C] No solutions

D] None of above

vi) Which of the following set is subspace of $\mathbb{R}^2$? [2]

A] W={ (x , y) / y=3x+3 }

B] W={ (x, y) / y = 0 }

C] W={ (x , y) / y = 3 }

D] W={ (x , y) / y = 3x-2 }

vii) Set of vectors $S = \left\{ v_1 = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}, v_2 = \begin{bmatrix} -6 \\ 3 \\ -9 \end{bmatrix}, v_3 = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix} \right\}$ is [2]

A] Linearly Independent set

B] Basis of $\mathbb{R}^3$

C] Linearly dependent set

D] Linear span of S= $\mathbb{R}^3$

viii) Let V be vector space of set of all polynomials of degree $\leq$ 3

V= $\{a_0 + a_1 t + a_2 t^2 + a_3 t^3 \ / a_0 \ , a_1, a_2, a_3 \in \mathbb{R}\}$ then Basis of V are [2]

A] Basis = $\{1 \ \ t \}$

B] Basis = $\{ \ t \ \}$

C] Basis = $\{0 \ , \ t \ , \ t^2 \}$

D] Basis = $\{ \ 1, \ t \ , t^2 \ \ t^3\}$

ix) Dimensions of the row space of the matrix $A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \\ 3 & 10 \end{bmatrix}$ are [2]

A] Row Space of A is Infinite dimensional

B] Dim Row Space A= 0

C] Dim Row Space A= 1

D] Dim Row Space A= 2

x) Basis of the Column space of the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 4 & 8 & -3 \end{bmatrix}$ are [2]

A] $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$

B] $\left\{ \begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 8 \end{bmatrix} \right\}$

C] $\left\{ \begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \end{bmatrix} \right\}$

D] $\left\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$

xi) Which of the following is Linear Transformation from $\mathbb{R}^2 \dashrightarrow \mathbb{R}^3$? [2]

A] T(x , y)= (x +y , xy , x)

B] T(x , y)= (x -y y, x)

C] T(x, y)= (x+y y+3 , y-x)

D] T(x, y)= (1+2y, x-3y)

xii) Consider the Linear Transformation A : $\mathbb{R}^3 \longrightarrow \mathbb{R}^3$ define as AX= Y [2]

Where A= $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ then dimensions of Im A are

A] Dimensions of Im A= 1
B] Dimensions of Im A= 2
C] Dimensions of Im A= 3
D] Dimensions of Im A= 4

xiii) Consider the Linear Transformation A : $\mathbb{R}^3 \longrightarrow \mathbb{R}^3$ define as AX= Y [2]

Where A= $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 9 \end{bmatrix}$ then Dimensions of Kernel A are

A] Dimensions of Kernel A = 1
B] Dimensions of Kernel A = 2
C] Dimensions of Kernel A = 3
D] Dimensions of Kernel A = 4

xiv) Linear Transformation Y = AX where A= $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ is [2]

A] Regular B] Orthogonal C] Singular D] Composite

xv) Linear Transformation Y = AX where A= $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 1 & 1 & 3 \end{bmatrix}$ is [2]

A] Regular B] Orthogonal C] Singular D] Composite

Q2 **Solve any two out of three**

a) Let V= $\mathbb{R}^3 = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \mid x, y, z \in \mathbb{R} \right\}$ with standard inner product defined as $\left\langle \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \right\rangle = x_1 y_1 + x_2 y_2 + x_3 y_3$ then by Gram-Schmidt orthogonalization process find orthogonal basis of the set of vectors S= $\left\{ \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \right\}$ [5]

b) Let V be a vector space of polynomials with inner product defined as $\langle f(t), g(t) \rangle = \int_{-1}^{1} f(t) g(t) dt$ then by Gram-Schmidt orthogonalization process find orthogonal basis of set of vectors $S = \{v_1 = 1 \quad v_2 = 1 + t, \; v_3 = t^2\}$ [5]

c)) Let V= $\mathbb{R}^3 = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \mid x, y, z \in \mathbb{R} \right\}$ with standard inner product defined as [5]

$\left\langle \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \right\rangle = x_1y_1 + x_2y_2 + x_3y_3$ then for the vectors

$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $v_2 = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$, $v_3 = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$ find

i) Projection of v_2 on v_1

ii) ii) $|| v_2 ||$

iii) III) Angle θ angle between vectors $v_1 \& v_3$

Q.3 **Solve any two out of three**

a) Find all Eigen values and Eigen vectors of the matrix A= $\begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$ [5]

b) Check whether A is diagonalizable & if yes diagonalize it where A= $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ [5]

c) Verify Caley-Hamilton Theorem & use it to find A^{-1} for the matrix A= $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ [5]

Q.4 **Solve any two out of three**

a) Find the symmetric matrix that corresponds to the following quadratic form and hence determine the nature of the quadratic form [5]

Q(x,y,z)=$2xy + 2xz + 2yz$

b) Find Signature of the quadratic form [5]

Q(x,y,z)=$-2x^2 + 2xy - 2xz - 2y^2 + 2yz - 2z^2$

c) Using orthogonal digitalization find Canonical form corresponding to quadratic form Q(x,y)=$14xy$ [5]

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