Total No. of Questions – [4]   Total No. of Printed Pages: 04

G.R. No.	

PAPER CODE	U111 – 201 A(REG)

MAY 2022 (INSEM+ ENDSEM) EXAM

F.Y. B. TECH. (SEMESTER - II)

COURSE NAME: Linear Algebra

COURSE CODE: ES10201A

(PATTERN 2020)

Time: [2Hr]   [Max. Marks: 60]

(*) Instructions to candidates:

1) **Figures to the right indicate full marks.**
2) **Use of scientific calculator is allowed**
3) **Use suitable data where ever required**

Q.1 **Solve the following**

i) Rank of the non-singular matrix of order n is [2]

A] Equal to n

B] Less than or equal to n

C] Equal to n -2

D] Equal to 0

ii) Solution of Homogenous equations 2x+3y =0, 6x+9y=0 [2]

A] only trivial solution x=y=0

B] Non Trivial Solution $x= -\frac{3}{2}t$, & y= t

C] x = 2t & y = 3 t

D] No solution

iii) For the following system of equations x +2y+3z=1 & 2x+4y+6z=2 has [2]

A] Unique solution

B] No Solution

C] Two free parameter solutions

D] three free parameter solutions

iv) Echelon form of the matrix A= $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 9 \end{bmatrix}$ is [2]

A] $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

B] $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

C] $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

D] $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

v) If A is Non-singular square matrix then Non Homogenous system of equation AX=B has [2]

A] Unique
B] Infinite solutions
C] Always Imaginary solution
D] No solution

vi) Which of the following set is subspace of $\mathbb{R}^2$? [2]

A] W={ (x , 2x+5) / x $\varepsilon\mathbb{R}$ }
B] W={ (0, 0) }
C] W={ (x , 3) / x $\varepsilon\mathbb{R}$ }
D] W={ (x , 2) / x $\varepsilon\mathbb{R}$ }

vii) Set of vectors $S = \left\{ v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ is [2]

A] Linearly Independent set but does not span $\mathbb{R}^3$
B] Basis of $\mathbb{R}^3$
C] Linearly dependent set
D] S is not a basis of $\mathbb{R}^3$

viii) Let V be vector space of set of all polynomials of degree $\leq$ 3 [2]

V= $\{a_0 + a_1 t + a_2 t^2 + a_3 t^3 \ / \ a_0, a_1, a_2, a_3 \in \mathbb{R}\}$ then Basis of V are

A] $\{1 \ \ t\}$
B] $\{ \ 1, \ t \ , t^2 \ \ t^3\}$
C] $\{0 \ , \ t \ , \ t^2\}$
D] $\{ \ 0, \ 1, \ t \ , t^2 \ \ t^3\}$

ix) Dimensions of the row space of the matrix A = $\begin{bmatrix} 1 & 3 \\ 2 & 6 \\ 3 & 10 \end{bmatrix}$ are [2]

A] Row Space of A is Infinite dimensional
B] Dim Row Space A= 2
C] Dim Row Space A= 3
D] Dim Row Space A= 0

x) Basis of the Column space of the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ are [2]

A] $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$

B] $\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$

C] $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$

D] $\{ [0] \}$

xi) Which of the following is Linear Transformation from $\mathbb{R}^2 \longrightarrow \mathbb{R}^3$? [2]

A] T(x , y)= (x +y , xy , x)

B] T(x , y)= (x -y y, x)

C] T(x, y)= (x+y y+3 , y-x)

D] T(x, y)= (1+2y, x-3y)

xii) Consider the Linear Transformation $A : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ define as AX= Y

Where $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ then dimensions of Im A are [2]

A] 1 B] 2 C] 3 D] 4

xiii) Consider the Linear Transformation $A : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ define as AX= Y

Where $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$ then dimensions of Kernel A are [2]

A] 1 B] 2 C] 3 D] 4

xiv) Linear Transformation Y = AX where $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is

A] Composite B] Orthogonal C] Singular D] Skew symmetric [2]

xv) Linear Transformation Y = AX where $A = \begin{bmatrix} 2 & 2 & 0 \\ 3 & 3 & 0 \\ 1 & 1 & 3 \end{bmatrix}$ is

A] Regular B] Orthogonal C] Singular D] Composite [2]

Q2 **Solve any two out of three**

a) Let $V = \mathbb{R}^3 = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \mid x, y, z \in \mathbb{R} \right\}$ with standard inner product defined as [5]

$\left\langle \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \right\rangle = x_1y_1 + x_2y_2 + x_3y_3$ then by Gram-Schmidt orthogonalization

process find orthogonal basis of the set of vectors $S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \right\}$

b) Let V be a vector space of polynomials with inner product defined as [5]

$\langle f(t), g(t)\rangle = \int_{-1}^{1} f(t)g(t)dt$ then by Gram-Schmidt orthogonalization process find orthogonal basis of set of vectors $S = \{v_1 = 1 \quad v_2 = 1+t, \ v_3 = t^2\}$

[5]

c)) Let V= $\mathbb{R}$ $= \left\{ \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \mid x, y, z, w \in \mathbb{R} \right\}$ with standard inner product defined as

$\left\langle \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} \right\rangle = x_1y_1 + x_2y_2 + x_3y_3 + x_4y_4$ then for the vectors

$v_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ -1 \end{bmatrix}, v_3 = \begin{bmatrix} 2 \\ 3 \\ 2 \\ -1 \end{bmatrix}$ find

i) Projection of v_1 on v_2

ii) $\| v_3 \|$

iii) Angle θ angle between vectors $v_1 \& v_3$

Q.3 Solve any two out of three

a) Find all Eigen values and Eigen vectors of the matrix A= $\begin{bmatrix} 0 & 5 \\ 5 & 0 \end{bmatrix}$ [5]

b) Check whether A is diagonalizable & if yes diagonalize it where A= $\begin{bmatrix} 7 & 1 \\ -9 & 1 \end{bmatrix}$ [5]

c) Verify Caley-Hamilton Theorem & use it to find A^{-1} for the matrix [5]
A= $\begin{bmatrix} 7 & 1 \\ -9 & 1 \end{bmatrix}$

Q.4 Solve any two out of three

a) Find the symmetric matrix that corresponds to the following quadratic form and hence determine the nature of the quadratic form [5]
Q(x,y,z)=$2x^2 + 2xy + 2xz + 2y^2 - 2yz + 2z^2$

b) Find Signature of the quadratic form Q(x,y,z)=$5x^2 + 2xz - 2y^2 + 5z^2$ [5]

c) Using orthogonal digitalization find Canonical form corresponding to quadratic form Q(x,y)=$2x^2 + 2y^2 - 8xy$ [5]

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