Total No. of Questions – [4]   Total No. of Printed Pages: 04

G.R. No.	

PAPER CODE	U111-201A(RE)

MAY 2022 (INSEM+ ENDSEM) EXAM

F.Y. B. TECH. (SEMESTER - II)

COURSE NAME: Linear Algebra

COURSE CODE: ES10201A

(PATTERN 2020)

Time: [2Hr]   [Max. Marks: 60]

(*) Instructions to candidates:

1) **Figures to the right indicate full marks.**
2) **Use of scientific calculator is allowed**
3) **Use suitable data where ever required**

Q.1 **Solve the following**

i) Rank of the matrix A of order 3X3 having all minors of order 3 zero and having nonzero minor of order 2 is [2]

A] Equal to 0
B] Equal to 1
C] Greater than or equal to 2
D] equal to 2

ii) Non zero $Solution\ of\quad x+y=1, 2x+3y=2\ is$ [2]

A] x=0 ,y= 0
B] x=0 ,y= 1
C] x=1 ,y= 0
D] No solution

iii) In solving the system of equations AX = B if $\rho(A)=3$ & $\rho([A:B\})=3$ and having 3 number of unknown variables. Then given system has [2]

A] No Solution
B] Unique solution
C] One free parameter solution
D] Two free parameter solutions

iv) Rank of the non-singular matrix A of order 4X4 is [2]

A] 4   B] 3   C] 2   D] 1

v) If A is non-singular matrix then Non Homogenous system of equation AX=B has [2]

A] Unique solution
B] NO solutions
C] Infinite solutions
D] No conclusion

vi) Which of the following set is subspace of $\mathbb{R}^3$? [2]

A] W= { (x, y ,z) / x=y & z=1 }
B] W= { (x, y ,z) / x + y+ z=0 & y + z=0 }
C] W= { (x , y, z) / x + y + z=1 }
D] W= { (x , y, z) / 2x+3y+2z=1 }

vii) Which of the following is true for the Set of vectors- [2]

$S = \{v_1 = 1, v_2 = t, v_3 = t^2\}$

A] Linearly dependent set of polynomial vector space
B] Linearly dependent and orthogonal set of polynomial vector space
C] Linearly Independent set of polynomial vector space
D] All of the above statements are true.

viii) Let V be vector space of set of all polynomials of degree ≤ 1 [2]

V= $\{a_0 + a_1 t \ / \ a_0 \ , a_1 \in \mathbb{R}\}$ then Basis of V are

A] $\{0\}$
B] $\{ t \}$
C] $\{0 , t \}$
D] $\{1 \ t \}$

ix) Dimensions of the row space of the matrix A = $\begin{bmatrix} 1 & 3 \\ 2 & 6 \\ 3 & 10 \end{bmatrix}$ are [2]

A] Row Space of A is Infinite dimensional
B] Dim Row Space A= 0
C] Dim Row Space A= 1
D] Dim Row Space A= 2

x) Basis of the Column space of the matrix A = $\begin{bmatrix} 1 & 2 & -1 \\ 4 & 8 & -3 \end{bmatrix}$ are [2]

A] $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$
B] $\left\{ \begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 8 \end{bmatrix} \right\}$
C] $\left\{ \begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \end{bmatrix} \right\}$
D] $\left\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$

xi) Which of the following is Linear Transformation from $\mathbb{R}^2 \longrightarrow \mathbb{R}^3$? [2]

A] T(x , y)= (x +y , xy , x)
B] T(x , y)= (x -y y, x)
C] T(x, y)= (x+y y+3 , y-x)
D] T(x, y)= (1+2y, x-3y)

xii) Consider the Linear Transformation A : $\mathbb{R}^3 \longrightarrow \mathbb{R}^3$ define as AX= Y [2]

Where A= $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ then dimensions of Im A are

A] 1 B] 2 C] 3 D] 4

xiii) Consider the Linear Transformation $A : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ define as AX= Y

Where A= $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 9 \end{bmatrix}$ then dimensions of Kernel A are [2]

A] 1 B] 2 C] 3 D] 4

xiv) Linear Transformation Y = AX where A= $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ is

A] Regular B] Singular C] Orthogonal D] Composite [2]

xv) Linear Transformation Y = AX where A= $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 1 & 1 & 3 \end{bmatrix}$ is

A] Regular B] Orthogonal C] Singular D] Composite [2]

Q2 **Solve any two out of three**

a) Let V= $\mathbb{R}^3 = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \mid x, y, z \in \mathbb{R} \right\}$ with standard inner product defined as [5]

$\left\langle \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \right\rangle = x_1y_1 + x_2y_2 + x_3y_3$ then by Gram-Schmidt orthogonalization

process find orthogonal basis of the set of vectors S= $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$

b) Let V be a vector space of polynomials with inner product defined as [5]

$\langle f(t), g(t) \rangle = \int_{-1}^{1} f(t)g(t)dt$ then by Gram-Schmidt orthogonalization process

find orthogonal basis of set of vectors $S = \{v_1 = 1 \quad v_2 = 1 + t, \ v_3 = t^2\}$

c)) Let V= $\mathbb{R}^3 = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \mid x, y, z \in \mathbb{R} \right\}$ with standard inner product defined as [5]

$\left\langle \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \right\rangle = x_1y_1 + x_2y_2 + x_3y_3$ then for the vectors

$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, v_2 = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}, v_3 = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$ find

i) Projection of v_2 on v_1

ii) $|| v_2 ||$

iii) Angle θ angle between vectors v_1 & v_3

Q.3 **Solve any two out of three**

a) Find all Eigen values and Eigen vectors of the matrix $A=\begin{bmatrix} 14 & -10 \\ 5 & -1 \end{bmatrix}$ [5]

b) Check whether A is diagonalizable & if yes diagonalize it where $A=\begin{bmatrix} 5 & 2 \\ 2 & 2 \end{bmatrix}$ [5]

c) Verify Caley-Hamilton Theorem & use it to find A^{-1} [5]
for the matrix $A=\begin{bmatrix} 7 & 1 \\ -9 & 1 \end{bmatrix}$

Q.4 **Solve any two out of three**

a) Find the symmetric matrix that corresponds to the following quadratic form and hence determine the nature of the quadratic form [5]
$Q(x, y, z) = -2x^2 + 2xy - 2xz - 2y^2 + 2yz - 2z^2$

b) Find Signature of the quadratic form [5]
$Q(x, y, z) = 2x^2 + 2xy + 2xz + 2y^2 - 2yz + 2z^2$

c) Using orthogonal digitalization find Canonical form corresponding to quadratic form $Q(x, y) = -4x^2 - 4y^2 + 6xy$ [5]

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