Total No. of Printed Pages: 03

PRN No.	

PAPER CODE	U313-2111 (RE)

December 2023 (REEXAM)

TY (SEMESTER - I)

COURSE NAME: NUMERICAL METHODS Branch: MECHANICAL COURSE CODE: MEUA31201

(PATTERN 2020)

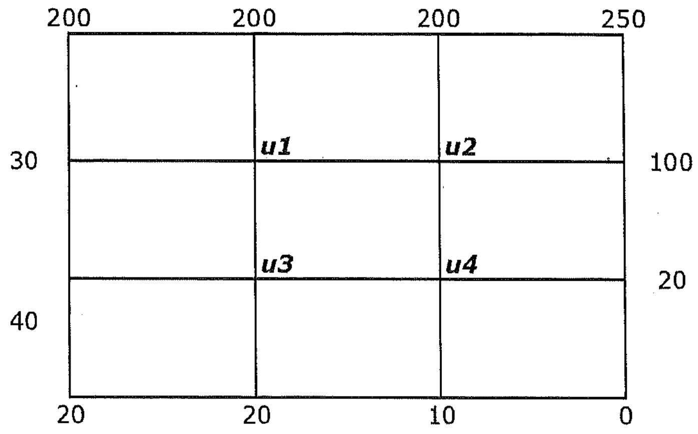
Time: [2 Hrs] **[Max. Marks: 60]**

(*) Instructions to candidates:

1) **Figures to the right indicate full marks.**
2) **Use of scientific calculator is allowed**
3) **Use suitable data wherever required**
4) **All questions are compulsory. Solve any two sub questions each from each Question 1 ,2, 3,4,5, and 6 respectively**

Q. No.	Question Description	Max. Marks	CO mapped	BT Level
Q.1	a) Find the root of a given equation $f(x) = e^x \cos(x) - 1.4$ using the Bisection method, Carry out computations upto 2nd stage. Assume root lies in the interval (3,5)	[5]	1	3
	b) Find the roots of equation using Newton Raphson method up to one interaction $x^3 - 2x - 5 = 0$, take initial guess 2	[5]	1	3
	c) The area of cross-section of a rod is desired up to 0.2% error. How accurately should the diameter be measured?	[5]	1	3
Q.2	a) Using the Gauss elimination method with partial pivoting, solve the equations: $18.0x + 10.0y + 22.0z = 78.0;$ $2.0x + 0.0y + 13.0z = 58.0;$ $19.0x + 20.0y + 21.0z = 85.0$	[5]	2	3
	b) Apply the Gauss-Seidal iteration method to solve the equations $1.3x + 0.8y + 0.3z = 25.0;$ $0.3x + 1.7y + 0.3z = 50.0;$ $1.0x + 0.5y + 2.0z = 20.0$ (take initial approximation = [0,0,0] and solution at 2nd stage)	[5]	2	3
	c) Apply the Gauss-Seidal iteration method to solve the equations $5.0x + 8.0y + 8.0z = 30.0$ $2.0x + 12.0y + 1.0z = 80.0$ $3.0x + 10.0y + 2.0z = 37.0$ (take initial approximation = [0,0,0] and solution at 2nd stage)	[5]	2	3
Q.3	a) Fit a straight line to the following data: x: 1.0, 2.0, 3.0, 4.0, 5.0 y: 8.0, 12.0, 14.0, 18.0, 20.0 *given* $\sum(x) = 15.000; \sum(y) = 72.000; \sum(x^2) = 55.000; \sum(xy) = 246.000$	[5]	3	3

<table>
<tr><td></td><td>b) Using Newton 's forward difference interpolation, find the value of $f(1.3)$, if
<table><tr><td>x</td><td>1.00</td><td>1.40</td><td>1.80</td><td>2.20</td></tr><tr><td>y</td><td>3.500</td><td>5.820</td><td>4.960</td><td>6.500</td></tr></table>
NFDI Formula : $y = y_1 + \frac{\Delta_{11}}{1!}u + \frac{\Delta_{12}}{2!}u(u-1) + \frac{\Delta_{13}}{3!}u(u-1)(u-2) + \ldots$</td><td>[5]</td><td>3</td><td>3</td></tr>
<tr><td></td><td>c) Fit a power equation ($y = ax^b$) to the following data:
<table><tr><td>x</td><td>61.0</td><td>26.0</td><td>7.0</td><td>2.6</td><td>1.2</td></tr><tr><td>y</td><td>350.0</td><td>400.0</td><td>50.0</td><td>600.0</td><td>200.0</td></tr></table>
given
$\sum(log(x)) = 4.540; \sum(log(y)) = 11.924;$
$\sum(log(x)^2) = 6.082; \sum(log(x)log(y)) = 10.995$</td><td>[5]</td><td>3</td><td>3</td></tr>
<tr><td>Q.4</td><td>a) Evaluate the given integral by Gaussian Quadrature 2-point method.
$$\int_0^1 \frac{dx}{x^2+1}$$</td><td>[5]</td><td>4</td><td>3</td></tr>
<tr><td></td><td>b) Evaluate the given integral by Simpson's 1/3rd rule.
$$\int_0^{\pi/2} e^{sin(x)}\, dx$$</td><td>[5]</td><td>4</td><td>3</td></tr>
<tr><td></td><td>c) Evaluate the given integral. (Assume n=m=4)
$$I = \int_0^2 \int_0^2 (x^2 + y^2 + 5)\, dx\, dy$$</td><td>[5]</td><td>4</td><td>3</td></tr>
<tr><td>Q.5</td><td>a) Employ Taylor's method to obtain approximate value of y at $x = 1.1$ for the differential equation $\frac{dy}{dx} = \log(xy)$, $y(1) = 2$.</td><td>[5]</td><td>5</td><td>3</td></tr>
<tr><td></td><td>b) Using the Runge-Kutta method of second order, solve for y at $x = 0.2$. From $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$, given $y(0) = 1$. (Take $h = 0.1$)</td><td>[5]</td><td>5</td><td>3</td></tr>
<tr><td></td><td>c) Using the 4th Order Runge-Kutta method, solve for y at $x = 0.1$. From $\frac{dy}{dx} = \frac{y-x}{y+x}$, given $y(0) = 1$. (Take $h = 0.1$)</td><td>[5]</td><td>5</td><td>3</td></tr>
<tr><td>Q.6</td><td>a) Classify the equation:
$$5\frac{\partial^2 z}{\partial x^2} + 6\frac{\partial^2 z}{\partial y^2} = xy$$</td><td>[5]</td><td>6</td><td>3</td></tr>
<tr><td></td><td>b) Given the values of $u(x,y)$ on the boundary of the square in the Figure 1, evaluate the function $u(x,y)$ satisfying the Laplace equation at the pivotal points of this figure by Gauss-Seidel method. (Show detail calculations for interaction no. 0, 1 and 2)</td><td>[5]</td><td>6</td><td>3</td></tr>
</table>

	 *Figure 1*			
	c) Solve the equation $\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2}$ subject to the conditions $u(x, 0) = sin(\pi x), \quad 0 \leq x \leq 1; u(0, t) = u(1, t) = 0$ using Crank-Nicolson method. Carryout computations for two levels, taking $h = 1/3, k = 1/36$	**[5]**	6	3

Note: [BT level- 1: Remember 2: Understand 3: Apply 4: Analyze 5: Evaluate 6: Create]